

Editorial: Optimisation methods of road pricing

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Toll charges are classified into congestion tolls (congestion pricing) and road tolls (private pricing) respectively, according to the purpose of pricing. Congestion pricing is used to mitigate network congestion by shifting traffic flow from peak periods to off-peak periods, from congested routes to less congested routes, or from private cars to public transport. Private toll pricing is used to recoup construction or maintenance costs of road links, when they are built fully or partially on private investment. This special issue reports some recent developments in road pricing. Three papers investigate congestion pricing, and one paper deals with toll adjustment for road franchising. The optimisation models and theoretical findings might be useful and meaningful for future research on road pricing.

Keywords: *congestion pricing; road toll; road franchising; subsidy*

1. The theoretical development of road pricing

Road pricing is implemented on road links for two different purposes. The first purpose is to mitigate network congestion by increasing the travel costs of some commuters, so as to shift traffic flow from peak periods to off-peak periods, from congested routes to less congested routes, or from private cars to public transport. The second purpose is to recoup construction or maintenance costs of road links, when they are built fully or partially on private investment. According to the aforementioned distinct purposes, tolls are classified into congestion tolls (congestion pricing) and road tolls (private pricing), respectively.

Congestion pricing on transportation networks has long been recognised as a subject of great importance from both a theoretical and a practical point of view. Particularly, since the successful implementation in London, congestion pricing stimulates the interests of researchers all over the world. The theory of congestion pricing dates back to Pigou (1920) and Knight (1924), who used the example of a congested road to make their points on externalities and optimal congestion charges. Later, the marginal cost pricing scheme, namely a toll equivalent to the difference between the marginal social cost and the marginal private cost is charged on each link in order to maximize the social net benefit, has been extended to general road networks (Walters, 1961; Dafermos and Sparrow, 1971; Dafermos, 1973; Smith, 1979a). In the case of fixed demand, the objective is to minimize the total travel cost spent on the network by the whole population; in the case of elastic demand, the objective is to maximise the total social welfare. Yang (1999) proved that the marginal cost pricing theory also applies to stochastic user equilibrium. Yang and Huang (1998) summarized the marginal pricing principle on the network in the presence of queues and delays.

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However, marginal cost pricing schemes cannot be implemented in practice in spite of their perfect theory. Firstly, such schemes have to face great pressure from public and political resistance. Secondly, the extremely high extra cost spent on the crew and equipment for toll collection in the entire network may not be acceptable. Thus many researchers started to investigate various forms of second-best pricing schemes where toll charges are levied only on a subset of links.

A few authors considered the second-best link-based congestion pricing that focuses on the determination of toll levels at predetermined locations. Yang and Lam (1996) presented a bi-level model considering fixed O-D demands, where the upper level program aims to minimise the total network travel time by trying different tolls on each link and the lower level program describes the route choice behaviour of network users. Verhoef (2002) proposed a mathematical approach to analyse the second-best toll strategies considering elastic demand. Yang and Zhang (2002) proposed bi-level models that explicitly incorporate the social and spatial equity constraints in terms of the maximum relative increase of travel costs between all OD pairs for various classes of drivers with different values of time. Yang and Zhang (2003) investigated the second-best link-based pricing schemes aiming at optimally selecting both toll levels and toll locations. Travel cost minimisation or social welfare maximisation with and without inclusion of implementation cost of toll charge is sought in general networks. Zhang and Yang (2004) studied the cordon-based second-best congestion-pricing problems on road networks, including optimal selection of both toll levels and toll locations. The road network is viewed as a directed graph and the cutset concept in graph theory is used to describe the mathematical properties of a toll cordon by examining the incidence matrix of the network. Maximisation of social welfare is sought subject to the traffic equilibrium problem with elastic travel demand. A mathematical programming model with mixed (integer and continuous) variables is formulated and solved by a combined use of a binary genetic algorithm and a grid search method for simultaneous determination of toll levels and cordon locations on the networks.

Some scholars developed models with alternative objective functions and constraints. Such objective functions or constraints might be authority's planning or management targets as well as economic, environmental or operational requirements, which can be characterised by side constraints of link flows. For the fixed demand traffic assignment problems, Bergendorff et al.(1997) and Hearn and Ramana (1998) drive the toll set of which each element is able to achieve minimum system cost, and the classical marginal cost toll vector is only one of them. Among the minimum system cost toll set, secondary objectives are sought, by minimising the revenue, the number of toll booths, or the maximum toll on any link. Similar results are found for the elastic demand traffic assignment problems by Hearn and Yildirim (2002).

Recently, Zhang and van Wee (2012) proposed a new objective function of maximising the reserve capacity of networks, and a new bi-level model is formulated for the implementation of congestion pricing, where either link-tolls or path-tolls are charged. To circumvent the difficulty of computing, the congestion pricing problem of simultaneous toll link and toll level optimisation is transferred into a single-level optimisation program with equilibrium constraints, and then the overall problem is formulated into a mixed-integer linear program for global optimum.

Some researchers consider the time-dimension of congestion tolls, namely time-varying congestion pricing. Analytical results are obtained with bottleneck models, where it is assumed that queues accumulate vertically (Arnott et al., 1993; Braid, 1989; De Palma et al., 2005; Kuwahara, 2007; Lai, 1994; Levinson and Rafferty, 2004; Mun, 1999). Commuters choose departure times by minimising the generalized travel cost including both queuing delay and unpunctuality penalty. Zhang et al. (2008) develops a model that simultaneously optimizes road tolls and parking fees for a system optimum in the day-long traffic commute. Pricing models that can consider the spatial dimension of queues have been developed by Lo and Szeto (2005), Verhoef (1999), and Verhoef (2001). In general networks, state-dependent marginal cost pricing

models are developed by Chow (2007), Huang and Yang (1996), and Yang and Huang (1997), based on optimal control theory.

Another area related to pricing is private road tolling, which often involves build-operate-transfer (BOT) contracts. The literature of private road pricing is much less than that of congestion pricing, although the practice of private road pricing is no less than that of congestion pricing, especially in developing countries. Verhoef and Rouwendal (2004) explores interrelations between pricing, capacity choice, and financing in transportation networks. It is shown that optimal congestion pricing and capacity choice over an entire network may cause user prices to increase more in initially mildly congested areas compared to heavily congested areas; and a flat kilometre charge under optimal capacity choice may result in first-best efficiency. Verhoef (2007) and Ubbels and Verhoef (2008) analysed capacity choice and toll setting by private investors in a competitive bidding framework organised by the government, where multiple criteria are considered and compared. Tan et al. (2010) investigates the properties of pareto-efficient BOT contracts using a bi-objective programming approach under perfect information. Efficient regulations are elucidated for both the public and private sectors to achieve a predetermined pareto-optimal outcome.

2. The contributions to this special issue

This special issue reports some recent works in the area of road pricing. The main aim is to expose the reader to the recent developments in optimisation methods of road pricing.

In the paper "**Finding second-best toll locations and levels by relaxing the set of first-best feasible toll vectors**", Ekström provides a framework of optimising both toll locations and toll levels in congestion pricing, with the objective of maximising social surplus. This optimisation problem is referred to as the toll location and level setting problem (TLLP) with binary decision variables. The non-convex and non-smooth TLLP is a hard problem to solve. A novel solution approach is provided which instead of directly solving the TLLP, makes use of the first-best toll level solution, in which no restrictions are imposed on toll locations or levels. Utilising the fact that the first-best toll levels on a route level are unique, the author tries to find the link toll levels which minimise the deviation from first-best route tolls. A mixed integer linear program is established, and for predetermined toll locations the resulting optimisation problem is a linear program. This approach is applied for two different network models, and results are provided to show the effectiveness of the methods. Furthermore, it has been shown that for the Stockholm network, virtually the first-best level of social surplus can be obtained with a significantly reduced number of toll locations.

Stewart and Ge's paper, "**Optimising network flows by low-revenue tolling under the principles of dynamic user equilibrium**", is intended to illustrate the feasibility of determining low-revenue toll sets to reduce the total network cost under dynamic user equilibrium (DUE) principles. Existing algorithms have been utilised for both the dynamic network loading and path-flow reassignment parts of DUE. The network loading part assumes a predefined flow profile whilst the path flow reassignment follows DUE, such that for any time t an individual driver may not benefit by changing route. The authors assume a fixed-toll level which applies to the whole time period being modelled; this removes the boundary problems inherent in tolling for a sub-period only. They seek to determine minimal or low-revenue tolls, by assigning one path to always have a zero-toll. It is proposed to develop the static heuristic of Stewart and Maher (2006) to move through possible paths in turn and incrementally increase the tolls one by one to seek a reduction in overall network cost. Finally, in the 4-link 3-path network, the capacity of each path is determined by the first link encountered; allowing bottlenecks to exist at a later position in the network will require full exploration. This would correspond to reducing the

capacity of link 2 in the 4-link 3-path network. The model used here is defined to allow tolls to be located only at the entry point to each path.

In the paper "**Pareto-improving toll and subsidy scheme on transportation networks**", Xiao and Zhang presents an original study on the economics of a link-based toll and subsidy scheme on a general transportation network. Different from a traditional congestion pricing scheme, the combination of toll and subsidy is found to be able to serve more planning purposes simultaneously, such as efficiency, fairness, and public acceptance. The authors first demonstrate that on a one-origin or one-destination network, a pareto-improving, system-optimal and revenue-neutral toll and subsidy scheme always exists and can be obtained by solving a set of linear equations. Recognising that such a scheme may not always exist for a multi-origin network, they then define the maximum-revenue problem with pareto-improving constraints to find the maximum possible revenue collected by the toll and subsidy scheme with optimal arc flows and non-increasing origin-destination travel costs. They discover that the problem is actually the dual problem of a balanced transportation problem, which can thus be solved efficiently by existing algorithms. At the end of the paper, a numerical example with a small synthetic network is provided for the comparison of the toll and subsidy scheme with other existing toll schemes in terms of OD travel disutilities.

The last but not the least paper "**Toll adjustment provisions for road franchising with unknown demand**", by Tan and Tan, proposes a revised trial-and-error ex post toll adjustment scheme to derive the Pareto efficient outcomes with unknown demand function. The authors point out that profit and social welfare are two main concerns in setting tolls in a road franchising scheme, and thus the pareto-efficient outcomes are preferred by both governmental and private sectors. Then the authors state that the error of the demand forecast or the unknown exact demand function is an important obstacle to set a pareto-efficient contract. Therefore a toll adjustment procedure is provided, and the procedure is proved to converge to a pareto-optimal solution that dominates the initial inferior toll scheme. The study also derives the necessary and sufficient conditions for the existence of the pareto-improving toll scheme. A numerical example is presented to demonstrate the application of the proposed schemes to an ex post contract adjustment for a BOT road project with unknown demand.

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